Tuning a Three-Element Windkessel Model

A beginner's guide, with a spreadsheet you can run

Companion files: `windkessel_tuning.xlsx` *(live spreadsheet). Every number in this guide is reproduced by that sheet.*

Contents

1 The big picture

When you simulate blood flow in an artery, you cannot model the entire circulation. You cut the domain somewhere downstream and ask: *what pressure does the rest of the body push back with?* The **three-element Windkessel model** is the most common answer. It replaces everything downstream of your cut with three numbers, and it is cheap enough to sit at the outlet of a full 3D simulation.

The name is German for "air chamber." Old fire engines used a sealed air pocket to smooth the pulsing of a hand pump into a steady stream from the hose. Your aorta does the same thing: it balloons during each heartbeat and recoils between beats, turning the heart's pulsatile ejection into steadier flow through the small vessels. The model captures that with an electrical analogy — pressure is voltage, flow is current — built from two resistors and one capacitor.

Why tuning matters

The three numbers are not measured directly; you *tune* them so the model reproduces a patient's (or a textbook's) mean pressure and pulse pressure. Get them wrong and your simulation's outlet pressure is wrong, which corrupts everything upstream. This guide shows the standard tuning loop and works one example end to end.

2 The three elements

The model takes the inflow waveform $Q(t)$ (volume of blood per second entering the arterial segment) and produces the pressure $P(t)$. Three parameters stand between them:

Symbol	Name	Represents	Units
R_c	Characteristic resistance	the proximal aorta's impedance	$\text{mmHg} \cdot \text{s/mL}$
C	Total arterial compliance	the arteries' springiness	mL/mmHg
R_d	Distal (peripheral) resistance	the small vessels	$\text{mmHg} \cdot \text{s/mL}$

In the circuit, blood first passes through R_c, then reaches a node where it can either charge the compliance C (stretch the arterial walls) or drain through R_d (flow out to the tissues). The sum $R_T = R_c + R_d$ is the **total peripheral resistance**.

Physical feel for each element

- R_T **(total resistance)** sets how high the pressure sits on average. More resistance, higher mean pressure.
- C **(compliance)** sets how much the pressure *swings* between beats. A stiff artery (small C) gives a big swing; a springy one absorbs it.
- R_c **(characteristic resistance)** shapes the sharp rise at the start of each beat. It is small — typically 5–10% of R_T.

3 The governing equation

Let $P_c(t)$ be the pressure at the node just past R_c (across C and R_d). Two facts define the circuit:

From the circuit to one clean equation

1. The compliance node. All the inflow Q arriving at the node either stretches the walls or drains through R_d:

$$Q = C\,\frac{dP_c}{dt} + \frac{P_c}{R_d} \quad\Longrightarrow\quad \boxed{\frac{dP_c}{dt} = \frac{Q}{C} - \frac{P_c}{R_d\,C}} \tag{1}$$

2. The series resistor. The same flow Q passes through R_c, so the aortic pressure is the node pressure plus one resistive jump:

$$\boxed{P(t) = P_c(t) + R_c\,Q(t)} \tag{2}$$

Equations (1)–(2) *are* the three-element Windkessel. Splitting it this way (solve the simple two-element model for P_c, then add R_cQ) avoids ever differentiating the flow, which is exactly why it drops straight into a spreadsheet.

Same thing in the textbook's single-line form

Substituting $P = P_c + R_cQ$ into (1) recovers the form you will see in papers:

$$\left(1 + \frac{R_c}{R_d}\right) Q + R_c\,C\,\frac{dQ}{dt} = \frac{P}{R_d} + C\,\frac{dP}{dt}. \tag{3}$$

It is identical — only messier to integrate, because of the dQ/dt term.

3.1 Marching it forward in time

With a time step Δt and steps $i = 0, 1, 2, \ldots$, the explicit (forward-Euler) update is what the spreadsheet does, one row per step:

$$P_c^{i+1} = P_c^{i} + \Delta t\left(\frac{Q^i}{C} - \frac{P_c^i}{R_d\,C}\right), \tag{4}$$

$$P^{\,i} = P_c^{i} + R_c\,Q^i. \tag{5}$$

Start from $P_c^0 \approx$ mean pressure and run several heartbeats until the waveform repeats (*periodic steady state*).

4 Which knob does what

Two clean relationships make tuning almost decoupled.

The two rules that make tuning easy

Mean pressure is set by resistance. Averaged over a heartbeat, the compliance neither gains nor loses volume, so mean inflow equals mean outflow. That gives

$$\text{MAP} = R_T\,\overline{Q}, \qquad \overline{Q} = \text{mean flow} = \text{cardiac output}. \tag{6}$$

Pulse pressure is set by compliance. The stiffer the arteries (smaller C), the larger the beat-to-beat swing $\text{PP} = P_{\text{sys}} - P_{\text{dia}}$. A useful upper-bound estimate is $\text{PP} \approx \text{SV}/C$ (stroke volume over compliance); the true value is somewhat smaller because blood drains during the beat.

Because (6) does not contain C, changing compliance barely moves the mean — it almost only changes the swing. That is what lets you tune the two targets one at a time.

5 The tuning loop

1. **Mean flow.** From cardiac output, or stroke volume × heart rate, get $\overline{Q}$.
2. **Total resistance.** Set $R_T = \mathrm{MAP}_{\mathrm{target}}/\overline{Q}$ from (6). This nails the mean pressure.
3. **Split the resistance.** Take R_c as 5–10% of R_T (or from the proximal characteristic impedance if you have it), then $R_d = R_T - R_c$.
4. **Guess compliance.** Start near $\mathrm{SV}/\mathrm{PP}_{\mathrm{target}}$.
5. **Simulate and read PP.** Run a few beats; read systolic, diastolic, and PP from the last cycle.
6. **Adjust C.** If PP is too large, *increase* C; too small, *decrease* it. Repeat step 5. The mean will hardly budge.

6 Worked example

Tune to 92/40 mmHg

Targets. Heart rate 75 bpm (period $T = 0.8$ s), stroke volume SV = 70 mL, ejection lasting $T_s = 0.3$ s. Target MAP = 92 mmHg and PP = 40 mmHg.
Step 1 — mean flow. $\overline{Q} = \mathrm{SV}/T = 70/0.8 = 87.5$ mL/s (that is 5.25 L/min).
Step 2 — total resistance. $R_T = 92/87.5 = 1.0514$ mmHg · s/mL.
Step 3 — split. $R_c = 0.05 \times R_T = 0.0526$, so $R_d = 0.9989$ mmHg · s/mL.
Step 4–5 — first guess. Try $C = 1.50$ mL/mmHg. Marching (4)–(5) for six beats gives a last cycle of

$$P_{\mathrm{sys}} = 114.7, \quad P_{\mathrm{dia}} = 74.0, \quad \mathrm{PP} = 40.6, \quad \mathrm{MAP} = 92.7 \text{ mmHg}.$$

Step 6 — nudge C. PP = 40.6 is a touch high, so increase compliance slightly to $C = 1.54$:

$$P_{\mathrm{sys}} = 114.4, \quad P_{\mathrm{dia}} = 74.5, \quad \mathrm{PP} = 40.0, \quad \mathrm{MAP} = 92.7 \text{ mmHg}. \checkmark$$

Result. $R_c = 0.053$, $R_d = 0.999$, $C = 1.54$ hit both targets. Note that MAP moved by less than 0.1 mmHg while C changed — exactly as (6) predicts. Final blood pressure: **114/74** mmHg.

The whole point of step 6 is that PP falls smoothly as C rises, while MAP stays put:

C [mL/mmHg]	1.30	1.40	1.50	1.60	1.70
P_{sys} [mmHg]	116.4	115.5	114.7	114.1	113.6
P_{dia} [mmHg]	71.7	73.0	74.0	75.0	75.9
PP [mmHg]	44.7	42.5	40.6	39.0	37.7
MAP [mmHg]	92.4	92.5	92.7	92.8	93.0

7 Using the spreadsheet

Open `windkessel_tuning.xlsx`. The left block holds your inputs; the yellow cell is the compliance C you tune. The columns to the right march the model forward in time, one row per step, using equations (4)–(5):

Column	Formula in words
t	step index $\times\ \Delta t$
Q	the half-sine inflow waveform
P_c	previous P_c plus the update (4)
P	$P_c + R_c\,Q$, equation (5)
last cycle	flag = 1 on the final heartbeat

The output cells read P_{sys}, P_{dia}, PP, and MAP off the last cycle. Change the yellow C and watch "PP − target" move toward zero. Change a target and the resistances retune themselves automatically.

8 Quick reference

Formulas

$$\begin{aligned}
\overline{Q} &= \mathrm{SV}/T = \text{cardiac output}\\
R_T &= \mathrm{MAP}_{\text{target}}/\overline{Q}\\
R_c &= (0.05\text{–}0.10)\,R_T, \qquad R_d = R_T - R_c\\
\frac{dP_c}{dt} &= \frac{Q}{C} - \frac{P_c}{R_d C}, \qquad P = P_c + R_c Q\\
\mathrm{MAP} &= R_T\,\overline{Q}, \qquad \mathrm{PP} \lesssim \mathrm{SV}/C
\end{aligned}$$

Quantity	Typical resting adult	Units
R_T (total resistance)	0.8–1.2	$\mathrm{mmHg \cdot s/mL}$
R_c (characteristic)	0.03–0.07	$\mathrm{mmHg \cdot s/mL}$
C (compliance)	1.0–2.0	mL/mmHg
MAP	70–100	mmHg
PP	30–50	mmHg

9 Common pitfalls

What trips people up

- **Reading the first beat.** The model needs several cycles to settle. Always read the *last* cycle, not the first.
- **Units.** Keep pressure in mmHg, flow in mL/s, time in s. Then resistance is mmHg · s/mL and compliance is mL/mmHg. Mixing mL/s with L/min is the most common error.
- **Trusting** $\text{PP} \approx \text{SV}/C$ **exactly.** It is only an upper bound; the real PP is smaller because blood drains during the beat. Use it to *start*, then tune.
- R_c **too large.** If R_c is more than about 10% of R_T, the pressure gets a spurious spike at the start of each beat. Keep it small.
- **Time step too big.** Forward Euler is only stable for $\Delta t \lesssim R_d C$. With $\Delta t = 0.01$ s and $R_d C \approx 1.5$ s you have plenty of margin, but do not use huge steps.